

# Giant proximity effect in a phase-fluctuating superconductor

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When a tunneling barrier between two superconductors is formed by a normal material that would be a superconductor in the absence of phase fluctuations, the resulting Josephson effect can undergo an enormous enhancement. We establish this novel proximity effect by a general argument as well as a numerical simulation and argue that it may underlie recent experimental observations of the giant proximity effect between two cuprate superconductors separated by a barrier made of the same material rendered normal by severe underdoping.

Josephson effect [1] – the ability of Cooper pairs to coherently tunnel between two nearby superconductors – represents one of the most spectacular manifestations of the electron pairing paradigm that underlies the BCS theory of superconductivity. If the barrier between the superconductors is formed by an insulating material (or a vacuum) the tunneling current is controlled by the overlap between the Cooper pair wave functions that extend into the empty space between the superconductors. When the barrier is made out of a normal metal (SNS tunneling) then the supercurrent can be much enhanced due to the proximity effect [2]. In essence, local superconducting order is induced inside the barrier which significantly enhances the distance over which pairs can tunnel.

The proximity effect is well understood and documented in conventional superconductors [3, 4]. In high- $T_c$  cuprates there now exists compelling experimental evidence for anomalously large proximity effect when the barrier is formed by an underdoped cuprate that would be in its normal state if studied in isolation [5, 6, 7, 8, 9, 10]. The critical currents of such junctions have been reported to exceed the expectations based on conventional theories by more than two orders of magnitude. While the early results raised some suspicion of being contaminated by various extrinsic effects, the most recent data [10] on very high quality films under closely controlled conditions leave little doubt that the effect is intrinsic and that it represents a qualitatively new type of Josephson tunneling. Previous theoretical attempts to elucidate this effect were mostly based on modeling inhomogeneous barriers using conventional mean-field methods [11, 12, 13] but they could not account for purely intrinsic effects.

In this Letter we formulate a theory of a new type of tunneling between two superconductors that occurs when the barrier is formed by an *unconventional normal metal*. The latter is characterized as a superconductor that has lost its phase rigidity due to phase fluctuations. According to one school of thought [14, 15, 16, 17, 18, 19] it is precisely this type of an unconventional normal metal that appears in the pseudogap state of cuprate superconductors [20] used as a barrier in the above experiments [8, 9, 10]. Recent experimental insights into the pseudogap phase [21, 22, 23, 24, 25] seem to confirm the existence of vortices well above the critical temperature, supporting the phase fluctuation paradigm and calling for a description of the Josephson tunneling processes in these exotic phases.

In the following we demonstrate, by a general argument and by extensive numerical simulations, that the Josephson tunneling in this situation (which we hereafter refer to as ‘SPS’ tunneling) is greatly enhanced compared to the SNS. We show that in SPS the dependence of the junction critical current on both the temperature and the barrier thickness is *qualitatively different* from the SNS case. The most striking difference is that in one particular regime we find *logarithmic* dependence of the junction critical temperature  $T_{\text{eff}}$  on the junction width  $d$ . At  $T < T_{\text{eff}}$  this allows the pairs to tunnel over vastly longer distances in accord with experiment.

The standard model describing a phase-fluctuating superconductor is defined by the XY Hamiltonian

$$H_{\text{XY}} = -\frac{1}{2} \sum_{\langle ij \rangle} J_{ij} \cos(\varphi_i - \varphi_j). \quad (1)$$

Here  $\varphi_i$  represents the phase of the superconducting order parameter on the site  $\mathbf{r}_i$  of a  $D$ -dimensional square lattice and  $J_{ij}$  are Josephson couplings between the neighboring sites  $\mathbf{r}_i$  and  $\mathbf{r}_j$ . Classical [26, 27] as well as the quantum [28, 29, 30, 31] versions of this model have been employed previously to study phase fluctuations in cuprates. Although quantum fluctuations may be important in cuprates, to demonstrate the effect in the simplest possible setting, we focus here on the effect of classical thermal fluctuations.

In the spatially uniform situation,  $J_{ij} = J$ , it is well known that the XY model undergoes a superconductor to normal transition at the critical temperature

$$T_c = cJ, \quad (2)$$

where we took  $k_B = 1$ . In 2 dimensions the transition is of the Kosterlitz-Thouless (KT) type [32], driven by the unbinding of vortex-antivortex pairs. The standard KT argument applied to the continuum XY model gives  $c = \pi/2$  while numerical simulations of the lattice model (1) yield  $c \simeq 0.93$  [33]. In order to model the proximity effect we consider the above XY Hamiltonian in the  $J$ - $J'$ - $J$  geometry illustrated in Fig. 1(a): two superconductors characterized by  $J_{ij} = J$  separated by a strip of width  $d$  and  $J_{ij} = J' < J$ . This configuration, at temperatures  $T'_c < T < T_c$ , behaves as a Josephson junction similar to the one studied in Ref. [10] since, according to Eq. (2), the strip should be in the normal state in this regime.

In this configuration the proximity of bulk superconductors will prevent vortex-antivortex pairs in the strip from unbinding at  $T'_c$ , leading to anomalously large proximity effect. In

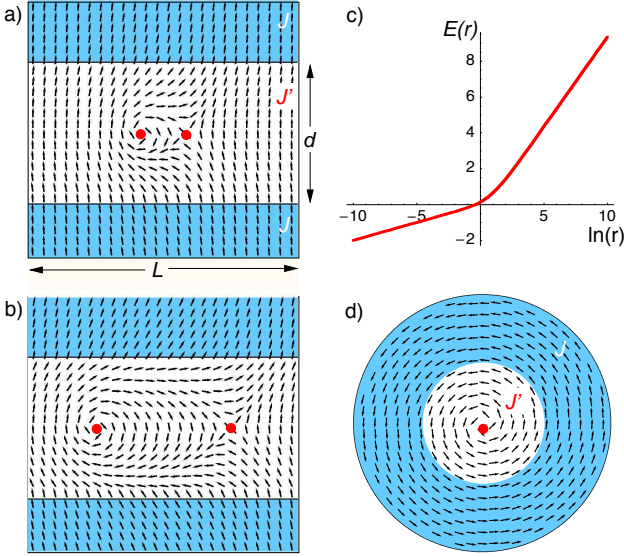


FIG. 1: (Color online) Superconducting phase distribution in the  $J$ - $J'$ - $J$  junction with  $J'/J = 0.3$  and a vortex-antivortex pair with size a)  $r = 0.3d$  and b)  $r = d$ . c) Energy of the vortex-antivortex pair, Eq. (4), in units of  $J$  as a function of the pair size in units of  $d$ , for  $J'/J = 0.2$ . d) Disk geometry used to estimate the vortex unbinding temperature.

order to see this consider a single pair of size  $r \ll d$  inside the strip. The phase gradient is largely confined to within the strip, Fig. 1(a), and thus the energy of such pair will be  $E_{va}(r) \simeq J' \ln(r)$ . Once  $r$  exceeds  $d$ , however, the phase gradient necessarily spills into the regions outside the strip, Fig. 1(b), and the pair becomes energetically more costly. In the limit  $r \gg d$  we expect  $E_{va}(r)$  to approach  $J \ln(r)$  and indeed this is borne out in a more detailed calculation summarized below. Since the KT transition occurs when  $r \rightarrow \infty$ , i.e. vortices become free, we may expect that the transition temperature in this geometry will be controlled by  $J$  and not by  $J'$ . The critical temperature and the critical current of such an SPS junction will thus be significantly enhanced.

Results presented in Fig.1 are based on the well known mapping [34] of the continuum version of Hamiltonian (1),

$$H_{XY} = \frac{1}{2} \int d\mathbf{r}^2 J(\mathbf{r})(\nabla\varphi)^2, \quad (3)$$

onto a 2D problem in electrostatics of point charges (representing vortices) in the dielectric medium characterized by a dielectric constant  $\epsilon(\mathbf{r}) \sim J(\mathbf{r})^{-1}$ . The phase configurations (related to the electric field vector) and the energy of the vortex-antivortex configuration can be obtained by the method of image charges. For vortex-antivortex pair lying on the symmetry axis of the strip the energy acquires a simple form,

$$E_{va}(r)/J' = \ln r + \sum_{j=1}^{\infty} \alpha^j \ln(1 + r^2/j^2), \quad (4)$$

with  $\alpha = (J - J')/(J + J')$  and  $r$  measured in units of  $d$ . It

is easy to verify that Eq. (4) indeed implies the asymptotic behavior stated above and shown in Fig. 1(c).

We now proceed to estimate the vortex unbinding temperature  $T_{\text{eff}}$  [32] in a system consisting of two superconductors characterized by  $J$  and  $J'$ . For simplicity we consider the disk geometry sketched in Fig. 1(d). The key advantage of this geometry is that, using the continuum Hamiltonian (3), we can calculate the energy of a vortex placed at the center of the disk exactly. By symmetry it is easy to see that  $|\nabla\varphi| = 1/r$  and it follows that the vortex energy is

$$E_v = \pi [J' \ln(d/\xi) + J \ln(L/d)], \quad (5)$$

where  $\xi$  is the vortex core cutoff and  $d, L$  are inner and outer radii respectively. We shall henceforth assume that (5) remains approximately valid even when the vortex is placed slightly off-center. The entropy of the single-vortex configuration can be estimated as  $S_v \simeq \ln(L/\xi)^2$  and the vortex unbinding temperature is then obtained by examining the free energy of the system  $F = E_v - TS_v$ . This yields

$$T_{\text{eff}} \simeq \frac{\pi}{2} J \left[ 1 - \left( 1 - \frac{J'}{J} \right) \frac{\ln(d/\xi)}{\ln(L/\xi)} \right]. \quad (6)$$

The above formula is physically reasonable: it interpolates smoothly between the limiting cases  $d \rightarrow \xi$  and  $d \rightarrow L$ , giving  $T_c = (\pi/2)J$  and  $(\pi/2)J'$  respectively, in accord with Eq. (2). On the other hand we do not expect Eq. (6) to remain accurate for  $J' \ll J$ . Indeed  $J' \rightarrow 0$  constitutes a singular limit: here we expect  $T_{\text{eff}} = T_c$  if  $d = 0$ , whereas  $T_{\text{eff}} = 0$  for all finite  $d$ . This discontinuous behavior is unlike the linear decrease of  $T_{\text{eff}}$  from  $T_c$  to  $T'_c = 0$  predicted by Eq. (6) when  $J' \rightarrow 0$ .

The key implication of the above estimate is that the critical temperature should scale with the ratio of *logarithms* of  $d$  and  $L$ . Such an unusual scaling is a direct consequence of the non-local nature of the phase field generated by a vortex and is much more general than the crude treatment presented above may suggest. Specifically, we shall establish below by detailed numerical simulations that the logarithmic scaling (6) also applies to the strip geometry of Fig. 1(a), and thus by extension, to the experimental setup of Refs. [8, 9, 10]. Since the critical current of a junction also scales with its  $T_c$  this establishes the advertised anomalous behavior of the SPS junction.

In order to validate the above considerations we now investigate the proximity effect systematically using numerical simulation. We employ a version of the Monte Carlo method in which we first map the XY Hamiltonian (1) onto a bond-current model [35] using a high-temperature expansion and then deploy the ‘worm algorithm’ [36], with only minor complications due to the inhomogeneity of  $J_{ij}$ . This method is well suited for our needs as it allows for efficient evaluation of the main quantity of interest, the helicity modulus  $\Upsilon$  and is known to circumvent problems due to critical slowing down near the transition.  $\Upsilon$  measures the response of the system to an externally imposed phase twist and its relevance follows from the fact that it is proportional to the critical current  $j_c$  the system can sustain before going normal [34].

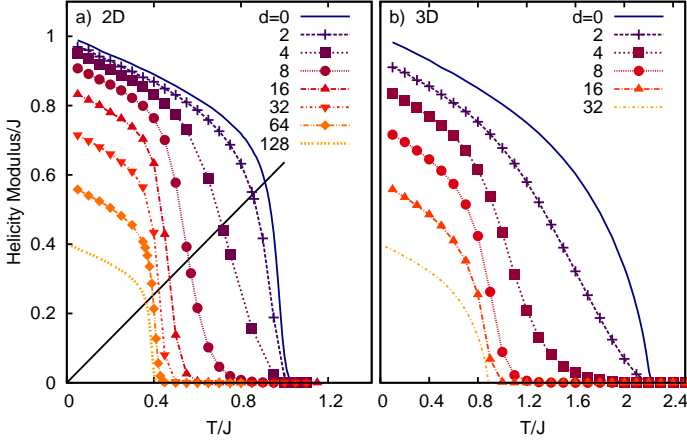


FIG. 2: (Color online) Helicity modulus as a function of temperature of the (a) 2D system with  $L = 128$  and (b) 3D system with  $L = 32$  and different sizes of the barrier  $d$  with  $J'/J = 0.4$ .

To check the validity of our algorithm, we first studied the homogeneous system with  $d = 0$  in 2D. Our results in this limit are in excellent agreement with those of Ref. [33] and, as  $L$  gets large, exhibit a clear approach towards the universal jump in  $\Upsilon(T)$  expected near the KT transition.

We consider next the 2D strip geometry illustrated in Fig. 1(a), for which we fix  $J'/J = 0.4$  and  $L = 128$  while varying the size of the junction  $d$  (here and hereafter we measure  $d$  and  $L$  in units of  $\xi$ ). The helicity modulus in the direction perpendicular to the junction is calculated as a function of temperature for a wide range of  $d$ . This is shown in Fig. 2(a). We observe a smooth evolution of  $\Upsilon(T)$  between the two homogeneous geometries ( $d = 0$  to  $d = L$ ). An interesting aspect of this data is the behavior of  $\Upsilon(T)$  at  $d \ll L$ : the helicity modulus (and thus  $j_c$ ) of the junction remains large at temperatures far exceeding  $T'_c$ . This is quite striking when one recalls that in this geometry all the supercurrent must pass through the region of small coupling  $J'$  which would be in the normal state at  $T > T'_c$  if studied in isolation. We may thus conclude that in the experimentally relevant regime  $d \ll L$  the junction critical current is controlled largely by the properties of the leads, as expected on the basis of heuristic arguments presented above.

Similar behavior occurs in 3D as illustrated in Fig. 2(b). We note that in 3D there is no universal jump at  $T_c$ ; instead our simulation recovers the expected continuous behavior characterized by the 3D-XY exponent  $\nu \approx 0.667$ . We note that both 2D and 3D results in Fig. 2 exhibit the characteristic *linear*  $T$ -dependence in the vicinity of the junction critical temperature that is ubiquitous in the experimental data [10].

In order to quantify the proximity effect we consider the junction critical temperature  $T_{\text{eff}}$  defined in 2D by the intersection of  $\Upsilon(T)$  with the line with slope equal to  $2/\pi$  [33]. In Fig. 3(a) the logarithmic dependence of  $T_{\text{eff}}$  on  $x = \ln d / \ln L$  expected from Eq. (6) is seen to hold for small  $x$ . The slope increases with decreasing  $J'/J$ , in accord with Eq. (6), although

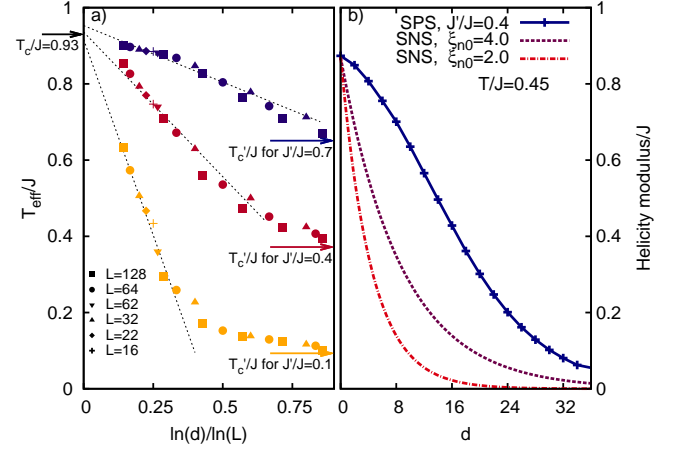


FIG. 3: (Color online) a) Junction critical temperature  $T_{\text{eff}}$  as a function of  $x = \ln d / \ln L$  for three different ratios  $J'/J = 0.7, 0.4$  and  $0.1$  in 2D. Finite size effects prevent these points from falling exactly on a smooth curve. The arrows indicate the expected  $T'_c$  values, reached when  $d \rightarrow L$ . Both  $d$  and  $L$  are measured in units of  $\xi$ . b) Comparison between SPS model and conventional SNS proximity effect.

it is quantitatively somewhat larger than predicted, presumably due to the differences between the strip geometry and the cylindrical geometry used to derive Eq. (6). For  $x \rightarrow 1$ ,  $T_{\text{eff}}$  tends to  $T'_c$ , as expected. In the limit  $J'/J \rightarrow 0$  we see a pronounced departure from the linear variation between  $T_c$  and  $T'_c$  predicted by Eq. (6). This is consistent with our expectation that Eq. (6) fails to describe this singular limit. Moreover, our numerical results demonstrate how the discontinuous jump of  $T_{\text{eff}}$  from  $T_c$  to  $T'_c = 0$ , expected for  $J' = 0$ , is approached continuously as  $J' \rightarrow 0$ .

We interpret the above numerical data as being in qualitative agreement with our heuristic picture of the giant proximity effect through a phase fluctuating superconductor in 2D. Notably, the agreement is excellent in the experimentally relevant regime of the narrow barrier  $x \ll 1$ .

In real superconductors the logarithmic interaction between vortices, which gives rise to the above phenomena, is cut off exponentially at length scales exceeding the magnetic penetration depth  $\lambda$ , or the effective ‘Pearl’ length  $\lambda_{\text{eff}} = \lambda^2/h$  in a 2D film of thickness  $h$ . When modeling a real superconductor one should thus replace  $L$  in all formulas by  $\Lambda = \max(\lambda, \lambda_{\text{eff}})$  except for very small junctions ( $L < \Lambda$ ) in which case  $T_{\text{eff}}$  will depend on the size of the macroscopic leads  $L$  as indicated by Eq. (6). In cuprates  $\kappa \equiv \lambda/\xi \approx 10^2 - 10^4$  and the barrier thickness  $d$  is typically of the order of 10-200 Å. Thus, the junctions of Ref. [10] are in the limit of relatively small  $x = \ln(d/\xi)/\ln \kappa$  and our considerations should apply.

In 3D point-like vortices are replaced by vortex loops. These lead to similar non-local phase gradients as vortex-antivortex pairs in 2D and it is thus to be expected that the enhancement of the proximity effect will persist in 3D SPS junctions. This is indeed confirmed by Fig. 2(b). Within the error bars our 3D numerical data hint at logarithmic dependence of  $T_{\text{eff}}$  on  $d$  similar to that in Fig 3(a) but we do not

currently have a simple heuristic picture for this dependence. Detailed account of our analysis will be given elsewhere [37].

Ref. [8] describes a Bi-2212/Bi-2201/Bi-2212 junction 123Å thick, with  $T_{\text{eff}} \approx 50\text{K}$ , over 3 times higher than  $T'_c \approx 15\text{K}$  of the Bi-2201 film. In the standard theory of SNS tunneling [2, 3, 4] the critical current

$$j_c \sim e^{-d/\xi_n}. \quad (7)$$

In the 2D clean limit  $\xi_n = \xi_{n0} \sqrt{T'_c/(T - T'_c)}$  with  $\xi_{n0} = \frac{1}{2}\xi(\Delta_0/k_B T_c)$  and  $\xi$  is the BCS coherence length of the order of tens of Å in cuprates. One thus expects essentially no supercurrent to flow through the above junction at temperatures significantly above  $T'_c$  according to the conventional theory. In the SPS scenario advocated in this Letter such enhancement is easily attainable due to the weak logarithmic dependence of  $T_{\text{eff}}$  on the barrier thickness  $d$ . Physically, this key difference stems from our assumption, rooted in extensive experimental evidence [14, 21, 22, 23, 24, 25], that underdoped cuprates above  $T_c$  behave as phase-disordered superconductors. To further exemplify this contrast we compare in Fig. 3(b) the  $j_c$  dependence on the junction width obtained from our model with the conventional SNS theory Eq. (7). We observe that for reasonable values of the BCS ratio ( $\Delta_0/k_B T_c \approx 4 - 8$  in cuprates) the SPS model implies vastly larger critical current than the conventional SNS theory. To the extent that our predictions can be systematically verified, experimental observation of the giant proximity effect can be viewed as a smoking gun evidence for the phase fluctuation paradigm.

Finally, it is worth pointing out that our results could also be relevant to suitably fabricated Josephson junction arrays as well as junctions of thin ferromagnetic films with easy-plane anisotropy and different exchange integrals (and thus, different Curie temperatures). These are also described by the XY model and our results apply unchanged, except that in ferromagnet the helicity modulus is replaced by the spin stiffness. Experimental measurements of its dependence on the thickness  $d$  of the inside layer provide another way to verify our predictions.

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